

IMPOSSIBILITY OF CONSTRUCTION

Authors: Aleksandre Museliani & Nikoloz Meskhi

Supervisor: Natia Miminoshvili

Georgian-American School, Tbilisi, Georgia, info@gas.edu.ge

1. Introduction

In plane geometry we often encounter problems about construction. Obviously, for construction we only use compass and unmarked straightedge. But the fact of impossibility of construction is unknown. The purpose of our work is to investigate this problem. As we have found out impossibility of construction problem has 2000-year history.

Geometric problems, which can be solved by using compass and straightedge, were always one of the most unique and beautiful parts of math. Even in ancient Greece active discussions about three classic problems occurred. These problems include:

1. Circle squaring
2. Angle trisection
3. Cube duplication

The ancient Greeks also knew that these problems did not have solution and was, therefore, impossible to answer all three above mentioned problems using only a compass and straightedge. Only in modern times, more than 2000 years later – they were formulated – were all three of these ancient problems proved insoluble using only compass and straightedge.

2. The main subject

Our work will partly focus on these issues, specifically whether or not it is possible to construct a triangle if there are given these elements

1. Only 3 medians (Medians case)
2. Only 3 heights (Heights case)
3. Only 3 bisector (Bisectors case)

In 1 and 2 cases Possibility of Construction is known and it is mentioned in many sources, but we have never seen the problem about constructing a triangle by three bisectors.

We had a hypothesis, that it is impossible and we began working on this problem.

Indeed, we proved that this hypothesis is true. We obtained

theorem:

It is impossible to construct a triangle with three bisectors using only straightedge (unmarked) and compass.

3. Conclusion

First of all, we can prove algebraically that, if there are given triangles three medians and three heights it is possible to construct this triangle. We have also presented construction's practical methods, but if we are given triangle three bisectors it is impossible to construct even an Isosceles triangle.

4. References

1. Courant R. and Robins H. "Geometric Constructions, Algebra and Number Fields" England: Oxford University Press, pp.117-164, 1996.

3. Neumann P.M. "Reflections on Reflection in a Spherical Mirror. " Amer, Math. Monthly 105, 523-528, 1998

4. Conway J.H. and Guy R.K. "There Greek Problems". New York: Springer-Verlag, pp.192-194, 1994.

5. Bold B. "Achievement of the Ancient Greeks" and "An Analytic Criterion for Constructability". New York: Dover, pp 1-17, 1982

6. Bogomolny A. "Angle Trisection" <http://www.cut-the-knot.org/Pythagoras/archi.shtml>.