COMBINATORIAL GAME WITH DOMINOES ON RECTANGULAR BOARDS

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1. Introduction

In this paper we study a variation of the game 'Domineering', which was popularized by Martin Gardner in 1974 [1]. For simplicity, we call it 'Domino tiling'. The game is played by two players, A and B, who alternate placing a domino on an empty space of the rectangular board. A player unable to make a move loses. There are no restrictions on the orientation of the domino, as long as it fits in the board.

2. Research methods

We use a number of methods to analyse the game. The first one is the strategy-based approach, on which we rely strongly throughout the project. The second one is the famous game *Nim* along with the devised techniques for solving it [2]. We also take advantage of the Sprague-Grundy Theorem and the Mex Rule [3].

The game *Nim* is played on several heaps of matchsticks. A move consists of strictly decreasing the number of matchsticks in one of the heaps. A player unable to make a move due to the lack of remaining heaps, loses.

3. Results

The results of this project are divided into five sections.

In the first section we analyse the planar version of the game. Firstly, we look at a $1 \times n$ strip. When n is even, there exists a strategy that allows A to win. When n is odd we use more complex methods – the Sprague-Grundy Theorem. Finally, we conclude that B can only win when $n = 1, 15, 35$ or $n \equiv 5, 9, 21, 25, 29 \ (mod\ 34)$. In all other cases A wins.

Figure 1 – Example with a $1 \times n$ strip.

Secondly, we see what happens when the board has at least two rows. We examine the game according to the parity of m and n. When both are even, B wins, using the board centre as a centre of symmetry. When m and n are of different parity, A can take the central 1×2 rectangle and then use it as a centre of symmetry.

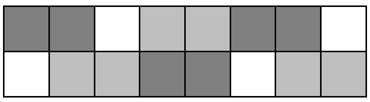

Figure 2 – Example with a $m \times n$ board.

In the next section, we examine a variation of the original game, where both players can place only horizontal dominoes. On a board $m \times n$ with m even, B wins. When the height m is odd, we can ignore the two big halves, as they place only horizontal dominoes, and examine only the middle row, which, in turn, reduces to playing the original *Domino tiling* on a $1 \times n$ strip.

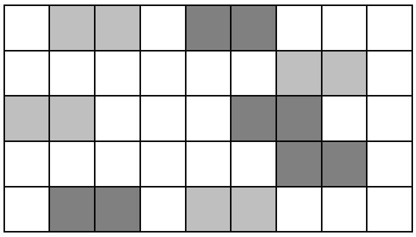

Figure 3 – Example with a $(2k + 1) \times (2p + 1)$ board.

In the third section, we generalize the results in multidimensional space. Firstly, we define the game in d-dimensional space. A *domino* we call a construction of two d-dimensional blocks $(1 \times 1 \times \ldots 1 \times 1)$ '*glued*' together. On a board with at least two even dimensions B wins, whereas on a board with exactly one odd dimension A has the upper hand.

The fourth section presents an analysis of the situation when we have a different type of domino. We consider the game in multidimensional space and instead of a domino we use a k-mino $(1 \times 1 \times 1 \times \ldots 1 \times k)$. Again, B wins if there are at least two even dimensions. However, A wins only on a board whose dimensions are of the same parity as the k.

Finally, we examine the game as a whole in terms of its relation to the game *Nim*. All previous results were derived using a strategy-based approach, which, however does not solve the entire problem. We again consider the game *Nim*. By adapting the approach used for the $1 \times n$ board, we prove that there exists a connection between our game and *Nim* for any board and any inserted figure (for example k-mino). We also show an efficient way of presenting a board as a graphical model, which can be used in a programme that computes the corresponding *Nim*-strip.

4. Conclusion

In our research we introduce a combinatorial game. We analyse small boards and perfectly solve the $1 \times n$ and $2 \times n$ boards. We find strategies for the $m \times n$ board. We examine a variation of the standard game and solve it completely. We generalize the results to multidimensional space as well as the type of domino. Finally, we successfully reduce our game to an already solved one, thus solving ours, too.

References

[1] West J., Championship-Level Play of Domineering, *Games of No Chance*, vol. 29, 1996.

[2] Conway J., On Games and Numbers, *A K Peters*, 2001.

[3] Siegel N., Misere Games and Misere Quotients, 2006.