MODELING OF TWO-DIMENSIONAL DIFFRACTION GRATING

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1. Introduction

The superposition of two mechanical waves can be constructive or destructive. In constructive interference the amplitude of the resultant wave at a given position or time is greater than that of either individual wave. This phenomenon finds its application in many fields, such is, for example the authentication of banknotes (Image 1). This makes crucial to be able to predict the diffraction pattern in advance. The goal of this project was to find a way to determine the coordinates of a two-dimensional diffraction pattern which can be observed on the wall when we two gratings perpendicular to each other and a monochromatic laser beam are used.

Image 1 - Determining the authenticity of Canadian banknote

2. Theory

If we imagine the rays issuing from row of slits (in a shape of double cone [1]) to be made visible by their intersection with a ground glass plate which is parallel to the row, the result would be a series of bright hyperbolas (Image 2) [2].

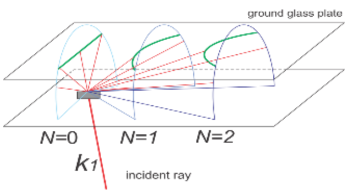

Image 2- Diffraction by linear grating shown on ground glass plate in physical space

If two perpendicular diffraction slits are used, double cones are perpendicular orientated too and bright dots are intersections between two hyperbolas. The equation of the hyperbola is:

$$\frac{x^2}{\left(\frac{l}{\tan(90°-\alpha)}\right)^2} - \frac{y^2}{l^2} = 1 \qquad (1)$$

where x and y are coordinates of point on hyperbola relatively to central maximum with coordinates (0,0), l is distance between slits and the screen and α is the angle of each diffraction maximum.

3. Results

Two gratings with the same number of closely-spaced set of slits (600 per millimeter) were used. The experimental setup shows the laser with 532 nm wavelength shining through gratings with the resultant image projected from about 9 centimeters away on a white screen (image 3).

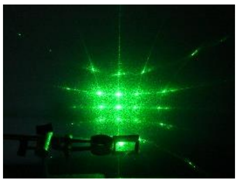

Image 3 - Diffraction pattern created by shining a laser through two perpendicular gratings

Then coordinates of some spots on the wall were compared with the theoretical coordinates that were calculated with the formula (1) (Table 1). The central spot has coordinates (0,0).

Table 1 – Comparison between experimental and theoretical value

spot	experiment		theory	
N	*x* [cm]	*y* [cm]	*x* [cm]	*y* [cm]
1	-4,2	7,2	-4,0	8,0
2	3,0	2,8	3,2	3,1
3	0,0	0,0	0,0	0,0
4	-8,0	-4,0	-8,2	-4,0
5	7,8	-4,0	8,2	-4,0

We can see that experimental and theoretical coordinates do not really differ a lot. The experiment confirms the theoretical procedure.

4.Conclusion

This work shows the direction of how to calculate the coordinates of bright spots of a two-dimensional diffraction pattern. If we use more gratings, the procedure is similar but reveals more complicated formulas for final solutions. We can determine a grating which creates various diffraction patterns. Furthermore, we can discover the equation of grating for every diffraction pattern. A practical way to use grating is for protection of money.

5. References

[1] Prosen, M. (2001). PRESEK: List za mlade matematike, fizike, astronome in računalnikarje. Ljubljana: DMFA.

[2] International Union of Crystallography. (1999). Got from http://www.iucr.org/publ/50yearsofxraydiffraction/full-text/principles, 4. 3. 2018