The Cherenkov effect: the correlation between the refractive index of a dielectric medium and the number of photons emitted

Daniel Stuefer
Dominicus College, Nijmegen, The Netherlands, d.e.stuefer@gmail.com

1. Introduction

Cherenkov radiation is emitted when a charged particle passes through a dielectric medium with greater velocity than the speed of light in that medium. One of the factors by which the emitted amount of radiation is influenced is the refractive index of the respective dielectric medium. When Cerenkov radiation is emitted, the photons are emitted at an angle θ, which is determined by the velocity of the charged particle and the refractive index of the dielectric medium. Those photons form a cone-shaped wave front. The aim was to investigate theoretically and experimentally whether a correlation exists between the refractive index and the amount of Cherenkov radiation emitted.

2. Research Methods

To find a theoretical correlation, several equations were combined. To test whether the predicted correlation between the nature of the dielectric medium (here: air pressure) and the amount of emitted radiation exists a detector was constructed using a 3D printer, a Fresnel lens and a Silicon Photomultiplier (SiPM) connected to an oscilloscope via a circuit.

3. Results

One aim was to show that the number of emitted photons depend on the refractive index, variation of which was simulated by changing air pressure.

First, the angle θ at which the photons are emitted had to be determined and can be given as

$$\theta(\beta, n) = \cos^{-1}\left(\frac{c}{vn}\right)$$

In this equation, n is the refractive index of the medium, v the velocity of the particle and c the velocity of light in vacuum. It was aimed at varying the pressure of the dielectric medium, air, in order to alter the refractive index. To assess how many photons to expect at a given pressure, the refractive index had to be calculated as:

$$n \approx 1 + \frac{\alpha p}{2 k_b T}$$

Here, n is the refractive index, α the polarization of the gas, T the temperature and k_b the Boltzmann's constant. The mean polarization of air equals $\alpha = 2.133 \times 10^{-29}\, m^3$. The temperature was set at room temperature, $T = 293.15\, K$.

$$n \approx 1 + \frac{2.133 \times 10^{-29}}{2 \cdot 1.381 \times 10^{-23} \cdot 293.15} \cdot p$$
$$= 1 + 2.635 \times 10^{-9} \cdot p$$

The correlation between the emitted photons, the charge of the particle and the emission angle, can be expressed as:

$$\frac{dN}{ds} = 490 \cdot z^2 \cdot \sin(\theta)$$

$\frac{dN}{ds}$ is the number of emitted photons per centimetre, z the charge of the particle, and θ the emission angle. As mostly muons and possibly some other particles with $|z| = 1$ were expected to be detected, z^2 was substituted with 1, θ with eq. 1, and $\frac{c}{v}$ with the value the muons were expected to have:

$$\frac{dN}{ds} = 490 \cdot 1 \cdot \sin\left(\cos^{-1}\left(\frac{1.0003}{n}\right)\right)$$

Furthermore, n was substituted with eq. 3:

$$\frac{dN}{ds} = 490 \sin\left(\cos^{-1}\left(\frac{1.0003}{(1 + 2.635 \times 10^{-9} \cdot p)}\right)\right)$$

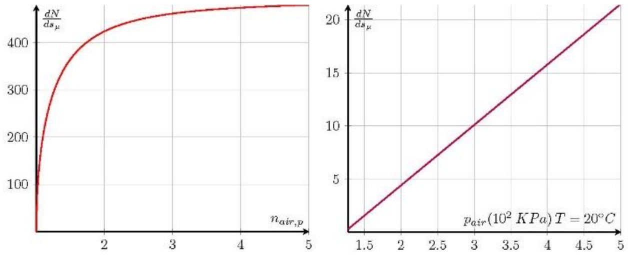

Figure 1: The number of emitted photons per centimetre a) as a function of n (Eq. 5) and b) as a function of p (Eq. 6).

The results revealed that with increasing air pressure the number of emitted photons increases.

4. Conclusion

With the assembled detector an attempt was made at measuring emitted photons under different pressure. However, emitted photons could not be identified, which could be due to several reasons. Firstly, there appeared to be a constant background noise signal, masking all other possible signals. This was most likely due to imperfect alignment of the circuit and/or electromagnetic signals interfering with the signals caused by emitted photons. Furthermore, the pressure in the detector could not be increased as it the detector was not airtight. Lastly, only a low percentage of the emitted photons during any given time would be detected due to the small detecting surface of the SiPM and an inefficient way of focusing the emitted photons.

References

Measuring the Molecular Polarizability of Air, M.J. Madsen, D.R. Brown, S.R. Krutz, and M.J. Milliman. Department of Physics, Wabash College, Crawfordsville, (July 7, 2009) [pages 1-5]