NEW PROPERTIES OF ISOSCELES TRIANGLE AND ISOSCELES TETRAHEDRON

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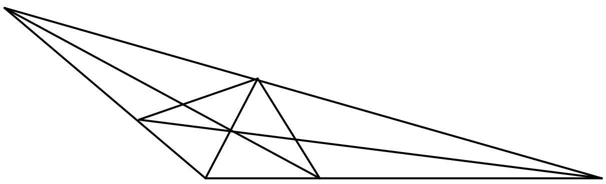
1.INTRODUCTION

When we take a look at geometry rules, we can see that triangle is isosceles if: 1) Two medians are equal, triangle is isosceles. 2) Two heights are equal, triangle is isosceles. But there is another condition, when we measure triangle by bisectors: 3) If two bisectors are equal, triangle is isosceles[1].

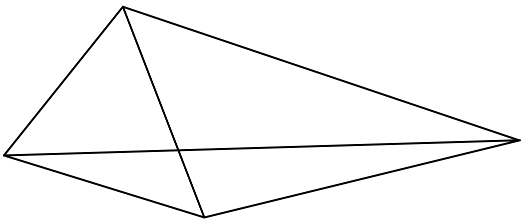
The main part of our project is a problem, which consists in "if two line segments formed from connecting the bases of triangle's bisectors are equal, is the first triangle isosceles or not?" In the second part of our project we investigated isosceles tetrahedron and found out some new properties. A particular kind of tetrahedron that arises in many contexts is the isosceles tetrahedron. (A tetrahedron is isosceles if the members of each pair of opposite edges are equal).

2. RESEARCH METHODS

In the first part of our project, by using algebraic and geometric methods in our work is found all such triangles i.e. There is exactly found all non-isosceles (scalene) triangles, in which the triangle formed by connecting bases of bisectors is isosceles.



Let's take a look at tetrahedron. The only way for all the faces of a tetrahedron to have the same perimeter, or to have the same area, is for them to be fully congruent. We shall show that equal perimeters, or equal areas, imply the tetrahedron is isosceles, from which the conclusion follows. The theorem on perimeters is almost immediate, while the theorem on areas is based on a special lemma.



3. RESULTS

Discovered in the first part, that in general case the first triangle will not be isosceles! But an author cannot find a triangle with this property.

For arbitrary x in interval

$$\left]1;\frac{1+\sqrt{17}}{4}\right[$$

exists non-isosceles (scalene) obtuse triangle with sides:

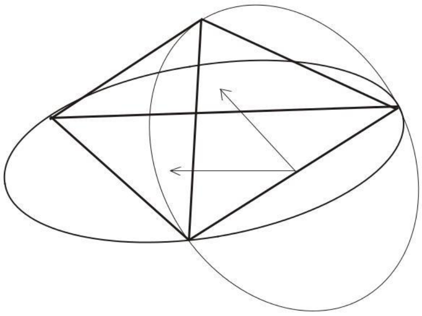
$$s;\, s\frac{x-\sqrt{D}}{2};\, s\frac{x+\sqrt{D}}{2};\, (s\in R^+)$$

Where

$$D=-\frac{2x^3+3x^2-4x-x}{2x+1}$$

For which, triangle obtained by connecting of bases of bisectors is isosceles.

In this case we proved the theorem: Tetrahedron is isosceles if and only if its inscribed and circumscribed spheres are concentric. Suppose the spheres, a plane cutting through a sphere has a circular intersection with the sphere and it divides the sphere into two segments. Spheres are concentric, consequently, the circles of intersection of the circumsphere and the planes of the faces are all the same size, this simply meant that the circumstances of the triangular faces are all the same sizes. Accordingly, the angles subtended by the edge at the other two vertices of the tetrahedron must be same. Therefore their sum is 180^0.



4. CONCLUSION

We solved isosceles triangle's and isosceles tetrahedron's problem with many attempts and found some new properties of isosceles triangle and isosceles tetrahedron. Now it's possible for everyone to have access to this properties and use it. As we told in the beginning there are many elementary properties of triangle and tetrahedron and ours' is one of them.

5. REFERENCES

1. H. S. M. Coxeter, S. L. Greitser // Geometry Revisited.
Toronto-New- York. 1967