Recurrence relation for congruum system solutions

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Definition 1. Positive integer r is congruent number, if the appropriate

$$y^2 = x(x^2 - r^2)$$

equation has unlimited numbers of rational solutions.
Definition 2. Positive integer r is congruent number if there exists three-term arithmetic progression, where the numbers are rational squares, with common difference r.

$$Xn - r = \square_1$$
$$X_n + r = \square_2$$

where $\square_1, \square_2$, and X_n are perfect squares of rational numbers. [1].
Definition 3. Positive integer r is a congruent number, if there exists right-angled triangle with rational sides and area equals to r.

Different sense of congruum problem
We have congruum problems in rational right triangle, when we can find rational right triangle a-b-c with area r.
We have Diophantine equation for congruent number, when legs of rational right triangle are given by using formulas:

$$a = \frac{2mn}{k} \quad b = \frac{m^2 - n^2}{k}$$

where m, n , k are positive integers (m>n) and number r is congruent number only if

$$k^2 r = mn(m^2 - n^2)$$

We have congruum problem in Elliptic Curve, when the elliptic curve:

$$y^2 = x(x^2 - r^2)$$

has a rational point.

Recurrence relation
We use definition 2 and for given system:

$$Xn - r = \square_1$$
$$X_n + r = \square_2$$

obtain new solution which is based on the previous one recurrence relation and is:

$$X_{n+1} = \frac{(X_n^2 + r^2)^2}{4X_n(X_n^2 - r^2)}$$

We can also have recurrence formulas for elliptic curve, rational right triangle and Diophantine equation. Like in congruum problem we can find out other solutions.

Conclusion
With help of one solution of congruum problem we can find out unlimited numbers of solutions.

References
1.Eric W.Weisstein
http://mathworld.wolfram.com/CongruumProblem.html