

On Approximations of Curves

Kirill A. Kuznetsov

Supervisor: Boris A. Zolotov

About work

It is often impossible to calculate the length of curve on a plane for sure – for example, if it is not known, which formula it is given by or by what principle it is constructed. On the other hand, it can be much easier to calculate the simple type curve length (for example, the length of broken line), which is close to initial one. One of the most important results of this paper is the possibility to estimate the accuracy of curve length approximation by curves length, which are close to initial curve.

Methods used in the work

The main goal is length estimation of curves – and used methods in the paper correspond the problem. For example, the curve length can be estimated by rating $|\gamma'|$, if the curve is regular. The simplest features of vector module and definite integral were used in the work. Also the concept of ε -approximation was involved, which was introduced in the paper.

Main results

Facts about curves were proved:

- For any ε curve has arbitrary long ε -approximations;
- If r - minimum radius of curvature and $\varepsilon < r/2$, then the length of any ε -approximations is does not exceed $(r-\varepsilon)/r \cdot L(\gamma)$;
- For the regular curve (or arbitrary rectifiable curve) its limit of infimums among ε -approximations length for ε striving to 0 is the length of initial curve
- The limit of shortest approximation, chosen among broken lines with vertical and horizontal edges and vertexes in $(1/n) \cdot Z \times (1/n) \cdot Z$ is $\int |x'| + |y'|$.

Conclusion

It was proven, that by given regular curve and ε you can build broken line, which occurs to be its ε -approximations. For this it is enough to take short edges of the broken line. This fact is useful when it is necessary to work with maps and object path in the nature.

References

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