# New explicit solution to the N-Queens Problem and the Millennium Problem

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## Introduction

The Millennium Prize Problems are seven problems in mathematics that were stated by the Clay Mathematics Institute in 2000. A correct solution to any of the problems results in a US $1000000 prize being awarded by the institute to the discoverer. Currently, the only Problem that has been solved is so-called Poincare conjecture. Another of these 7 Problems is related to the complexity of algorithms. Among these algorithms, polynomial algorithms are highlighted. The class of these algorithms is designated by P. Another class of algorithms are the algorithms which are able to check in polynomial number of steps that an answer is indeed a solution to a problem. Class of these algorithms is designated by NP. The Millennium Problem is the P versus NP problem. In August 2017 a group of Scottish mathematicians proved that N-Queens Completion Problem is NP-complete. Namely, if this problem can be solved in polynomial time, then P is equal to NP.

My work is devoted to the N-Queens Problem i.e. the problem of placing N chess queens on an $N \times N$ chessboard so that no two queens attack each other.

Famous German mathematician C.F. Gauss found 72 solutions for N=8. But 24 years later J.W.L. Glaisher proved using a method of determinants that for N=8 there are exactly 92 solutions. The existence of a solution for arbitrary N was proved by different authors using different methods. Now the number of different solutions Q(N) is computed only for 4<=N<=27. Calculation of Q(N) is related to the N-Queens Completion Problem (if we have m<N queens on the board, is it possible to complete this board to the solution of the N-Queens Problem). Existing methods of solving the completion problem stop working for N>=1000.

In this project, I try to find such an algorithm.

## Research methods

- Theoretical proofs. I introduce a brand-new way for representation of solutions that make proofs simpler than existing proofs of being arrangement a solution. To prove the results, I mostly use properties of inequalities and combinatorial constructions.
- Programming. The best way to check that an algorithm that I suggest works values of N up to 27 (or more) is the program.

## Results

A composition of A and B is a queens' arrangement obtained by insertion of arrangement B into queens' positions of arrangement A. I find the criterion of being a composition of solutions a solution: a composition of solutions A and B is a solution if and only if

1) $\{B(i) - i \mid 1 \le i \le |B|\} = \mathbb{Z}_{|B|}$.
2) $\{B(i) + i \mid 1 \le i \le |B|\} = \mathbb{Z}_{|B|}$.

Sufficiency was proved 100 years ago, and I prove the most difficult part - the necessity with new consequences.

Queen function with width k is a special almost usual linear map which is defined on a partition of a segment [1, N] by the k segments. I prove that for any N>3 there exists a solution which can be represented as a Queen function with width fewer or equal to 3. And this estimate of width cannot be reduced (proved with a program).

Based on obtained results I formulate a hypothesis: if for N-1 and N there are no solutions which are compositions of smaller boards then there exists a set of fundamental solutions which can be represented as a queen function with width fewer or equal to 3.

## Conclusion and future research

If this hypothesis is true then for such N I can construct a polynomial-time algorithm which solves N-Queens Completion and, consequently, Millennium P vs NP Problem.

The hypothesis raises a new problem: is the number of such N is infinite? In particular, is the number of primes of the form $2^k 3^l - 1$ is infinite? Or is the number of prime numbers $p$ such that $2p + 1$ is prime too is infinite?

## References

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