Maximal commutator length in a free group

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Introduction

Objects g and h are called commuting if gh=hg. A Group is commutative if any group elements are commuting. Any commutative finitely generated group can be presented as finite products of cyclic group. Commutative finitely generated group theory is easy to understand when finitely generated group theory is unobvious and very important. Methods of determination of extension of one group from commutative is an important in finitely generated group theory. Especially notions of commutator and commutator subgroup.
Commutator of elements g and h is $g^{-1}h^{-1}gh$ and it equals 1 when and only when g and h are commuting. Commutator subgroup or derived group is a group generated by all commutators.
By definition we obtain that any element from commutator subgroup can be presented as product of commutators. So we will call commutator length the minimal amount of commutators in such presentation. Free group is especially important it group theory. And we explored commutator lengths of words in free group. Main motivation for us is A.G.Myasnikov and O.G.Harlampovich theorem about existing such word that it's commutator length is bigger than commutator length of its square. We are interested in searching for this word.
This work is devoted to describing and realizing algorithms that helps to find all the members of every automorphisms group action of free group orbit and compute it's commutator length. Our main result is the table with results of such calculations.

Research methods

- Theoretical proofs. We used some theorems from scientific works to colligate words in commutator subgroup to make calculations easier
- Programming. We used Ocaml programming to generate list of words, reduce them, find their normal forms and then count their commutator lengths

Results

- We made some programs based on the theoretical part of the project and calculated commutator lengths for all normal forms of words from commutator subgroup.

Conclusion and future research

Further work in this direction can be presented as adding some improvements to algorithms and to the programs' codes. Another way is to try to create the same programs for bigger sets, for example for alphabets of three or more elements.

References

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