THE DYNAMICS OF THE GEE-HAW WHAMMY DIDDLE

Rachel Qing Pang
Raffles Institution (Junior College), Singapore

1 Introduction

The Gee-Haw Whammy Diddle is a mechanical toy that has fascinated many with its apparent ability to convert linear motion into circular rotation. The dynamics of this toy appear deceptively simple, but is actually at the centre of much debate and discussion today. Further investigation reveals a complex interplay of multiple processes, namely the rod vibration, nail elliptical motion, and propeller rotation. The aim of our research is to explain the phenomenon, and to investigate some counter-intuitive effects of certain parameters on the steady state angular velocity of the propeller.

2 Mathematical Treatment

We aim to conduct a full, in-depth investigation of the phenomenon which may not be found in existing literature. The subsequent sections discuss the physics of the system, which has been dissected into its three main components: rod vibration, nail elliptical motion, and propeller rotation.

2.1 Rod Vibration

Instead of modeling the rod as a typical cantilever beam with a mass per unit length, we suggest modelling it as a massless beam with an effective mass at its end. This is for the purpose of enabling easy variation and calculation of the rod's natural frequency ω, via the addition of masses to the end of the rod.

$$\omega = \sqrt{\frac{\frac{3EI}{L^3}}{\frac{3m}{1.875^4}+M}} \quad (1)$$

Where m is the mass per unit length of the rod, E is the experimentally-verified Young's modulus, I is area moment of inertia, and L is length. 1.875 is the first solution of $\cosh x \cos x = -1$, which corresponds to the coefficient for the first, and dominant, mode of vibration of the rod.

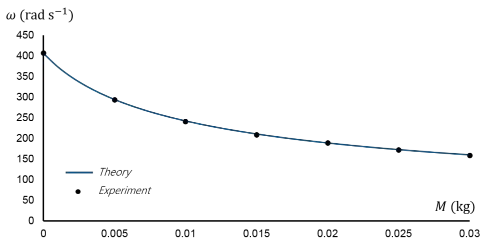

Figure 1: Calibration graph of experimentally and theoretically-determined ω with varying M.

2.2 Nail Elliptical Motion

Additional tension was introduced along one diagonal of the rod to introduce asymmetry into the system. This increases the vibration frequency of the rod along that diagonal, allowing the propeller to rotate in different directions.

3.3 Propeller Rotation

We model the rotation of the propeller as an effect of the nail's elliptical motion. By Newton's Second Law of Motion, we describe propeller rotation in the tangential and radial directions respectively.

$$d\ddot{\varphi} + 2\dot{d}\dot{\varphi} = \frac{f}{m_P} + (\ddot{x}\sin\varphi + \ddot{y}\cos\varphi) + g\cos\varphi \quad (2)$$

$$\ddot{d} - d\dot{\varphi}^2 = -\frac{N}{m_P} + (-\ddot{x}\cos\varphi + \ddot{y}\sin\varphi) + g\sin\varphi \quad (3)$$

The Reynolds numbers of the propellers used were calculated using $R_e = \frac{\rho v l}{\mu}$, and were found to be in the turbulent flow regime, giving the torque equation,

$$I_P\ddot{\boldsymbol{\theta}} = -c\dot{\theta}^2\hat{\boldsymbol{\theta}} - R\boldsymbol{f} \quad (4)$$

When the propeller is rolling around the nail under a no-slip condition, the angular velocity of the propeller is related to that of the nail by

$$\dot{\theta} = \frac{R-r}{R}\dot{\varphi} \quad (5)$$

3 Experiment

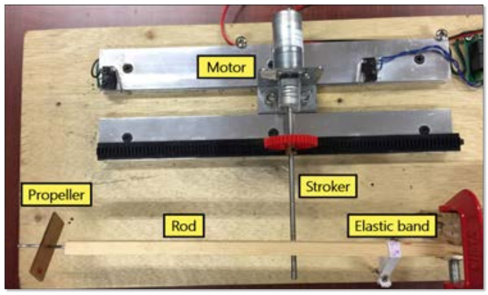

Figure 2: Labelled image of experimental set-up.

The experimental set-up was specifically made to model the conventional operation of a Gee-Haw Whammy Diddle by a human.

4 Results and Discussion

Numerous relevant parameters were varied and their experimental results were confirmed using a numerical simulation, some of which are shown below.

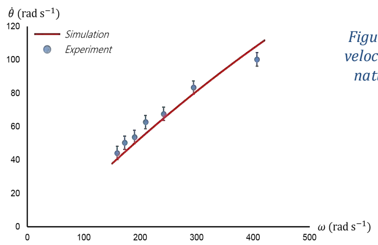

Figure 3: Graph of angular velocity of propeller against natural frequency of rod.

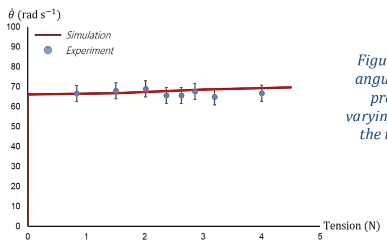

Figure 4: Graph of angular velocity of propeller with varying tension along the tied diagonal.

5 Conclusion

Our work aims to present an in-depth explanation of the toy that is confirmed by the variation of relevant parameters. The corroboration between the numerical simulation and experimental verification for the variation of multiple different parameters provides a degree of validation for our theoretical model.