

# EXCIRCLES AND EXSPHERES, ALSO POINTS WHICH ARE CONNECTED WITH THEM

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## 1. Introduction

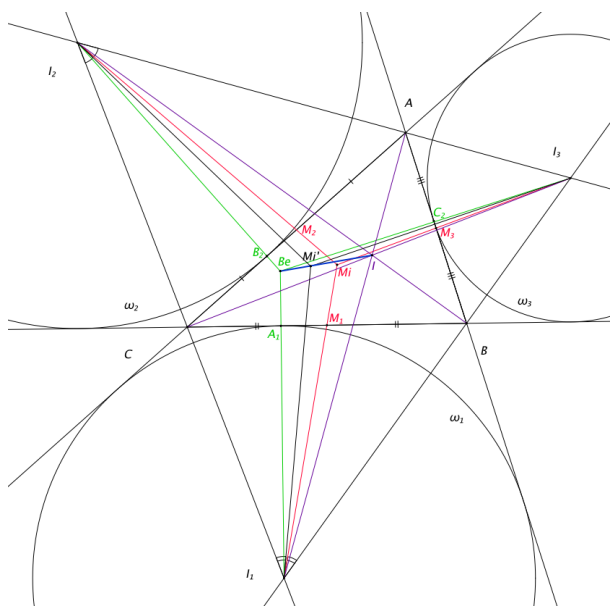
The German scientist Christian Gerni von Nagel was the first person, who started to explore points connected with excircles. At the same time scientists learned about exspheres in triangle pyramid. Nowadays topic is actively used in Olympiads of different level. The main goal of the work was a systematization of knowledge about the points connected with excircles and exspheres and creating new materials on this topic.

## 2. Methods

Work includes author's problems and theorems, what proves that fact that topic is a huge field for creating new problems, which could be used in the mathematical clubs or competitions, for developing students savvy. For example,

### Theorem 1

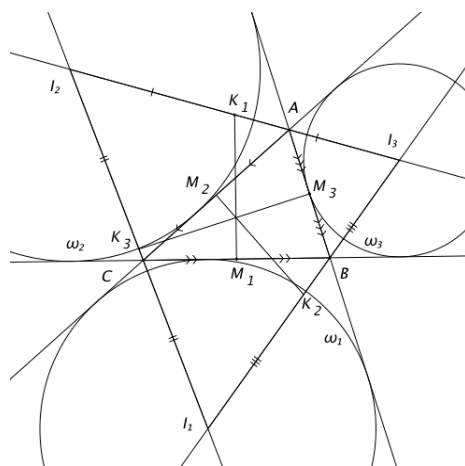
In triangle  $ABC$  3 triangle centers: incenter ( $I$ )<sup>1</sup>, Bevan's point ( $Be$ )<sup>2</sup> and point which is isogonal conjugated with Mittenpunkt ( $Mi$ )<sup>3</sup> point — belong to one line (pic. 1)



Pic. 1

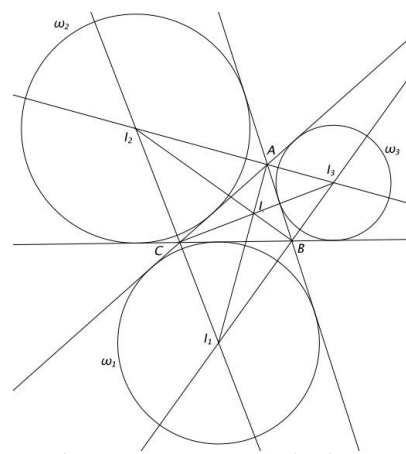
### Theorem 2

Let middles of the line segments  $I_1 I_2$ ,  $I_2 I_3$ ,  $I_3 I_1$  be  $K_1$ ,  $K_2$ ,  $K_3$  respectively, where  $I_1$ ,  $I_2$ ,  $I_3$  are centers of excircles in triangle  $ABC$ .  $M_1$ ,  $M_2$ ,  $M_3$  are middles of triangle sides  $BC$ ,  $CA$  and  $AB$  respectively (pic.2). Lines  $K_2 M_2$ ,  $K_1 M_1$ ,  $K_3 M_3$  intersect each other in one point.



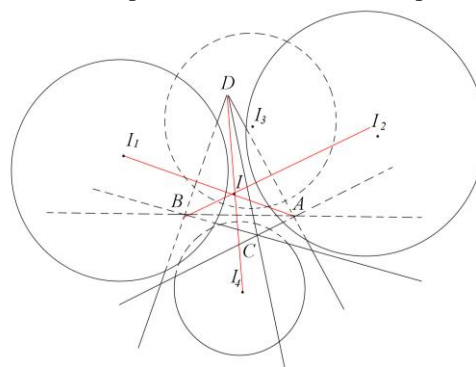
Pic. 2

Also in the work famous points connected with excircles in the plane were matched with points connected to exsphere in space. For example, lines, which are connecting vertexes of triangle with centers of relevant excircles, intersect each other in one point — center of inscribed circle of triangle, incenter (pic.3).



Pic. 3

Analogically, lines, which are connecting centers of exspheres with vertexes of triangle pyramid, intersect each other in one point — center of inscribed sphere (pic.4). [2]



Pic. 4

## 3. Conclusion

In the work knowledge about topic was systematized. Points connected with excircles were matched with ones connected with exspheres. In addition, author's problems were created for expanding the material in this topic. Also author's solutions, which are shorter than those before, were illustrated. Moreover, pedagogical technique that helps in the learning this topic was developed.

## 4. References

1. Kimberling, C. "Triangle Centers and Central Triangles." Congr. Numer. 129, 1-295, 1998.
2. Panarin Y.P. "Elementary geometry – part 2" – MCCME

<sup>1</sup> Incenter — point of intersection of bisectors in triangle

<sup>2</sup> Bevan's point — point of intersection of lines which are connecting vertexes of triangle and points of touching relevant excircles with sides of triangle.

<sup>3</sup> Mittenpunkt point — point of intersection of lines which are going through centers of exspheres and middle of relevant side sides of triangle. [1]